PAPERS IN PHYSICS, VOL. 17, ART. 170001 (2025)

www.papersinphysics.org

Received: 20 March 2025, Accepted: 15 May 2025
Edited by: F. Queiroz, International Institute of Physics in Natal, Brazil

DOI: https://doi.org/10.4279/PIP.170001

ISSN 1852-4249

# On an interacting ghost dark energy model in a non-flat universe with accelerating expansion

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An interacting ghost dark energy with experimental constraint in a non-flat universe with accelerating expansion is studied in this report. Based on the linear relationship between dark energy and the Hubbard function, it is found that the non-gravitational interaction between dark energy and cold dark matter near a = 0 leads to the transfer of energy from cold dark matter to dark energy. The cosmological evolution of the interacting ghost dark energy density, its equation of state and the deceleration parameters in a non-flat universe are obtained analytically. Based on the astronomical observations on the present value of the energy density of state $\omega_{D0}$, we impose restrictions on the interaction factor. It is found that there is a simple reverse linear relationship between $\omega_{D0}$ with the interaction factor, and all the deceleration parameters $q_0 \subset [-0.91, -0.38]$ are negative, which aligns with the astronomic observation on the accelerated expansion of the universe.

## I Introduction

The astronomical observations regarding type Ia supernovae [1, 3], Large Scale Structure [4, 5] and Cosmic Microwave Background anisotropies [6] have convincingly substantiated the accelerated expansion of the Universe. So far, cosmologists have proposed many theoretical models, and Dark energy is that which is most accepted as an explanation ofthe expansion of the universe. Dark energy must meet the condition that its pressure be negative, which leads to an accelerated expansion of universe. The most obvious candidate for explaining the nature and origin of dark energy is the cosmological constant [7,8] which has the equation of state $\omega = -1$. However, this explanation itself inevitably leads to further difficulties, such as the fine-tuning problem, etc. For that reason, a series of alternative theoretical models have been proposed. In particular, a group of scalar field dark energy theories including quintessence [9–14], K-essence [15, 16], tachyon [17–20], phantom [21], ghost condensate [22, 23] and quintom [24–29], braneworld models [30] and Chaplygin gas models [31], etc., have been widely studied. Further studies suggest that dark energy contributes around 70% of the total energy density of the Universe. The rest of the contribution is predominated by another exotic component, called "dark matter" (roughly around 26%) [32]. However, the origin, nature and dynamics of dark energy are absolutely unknown even after a series of observational missions running over the last twenty years. In order to understand the dynamics of our Universe, in particular the dynamics of the dark sector, the non-gravitational interaction function $Q$ between dark energy and cold dark matter is introduced under the condition that the conservation of total energy density is satisfied to form the so-called interaction

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dark energy model or coupled dark energy model [33–40], in which the equation of state is assumed to be a constant and the positive value of $Q$ indicates that there is an energy transfer from the cold dark matter to dark energy, while for negative $Q$, the reverse scenario ocurrs, meaning that energy is transferred from dark energy to cold dark matter; thus, $Q$ was named the rate of energy transfer. Supriya Pan et al. also gave the field theory analysis of the interaction dark energy model [38].

In this report, we will consider the non-gravitational interaction between cold dark matter and dark energy based on the view that dark energy originates from ghost fields, and obtain the analytical expressions of the energy density of cold dark matter and dark energy, the equation of state of dark energy, and the deceleration parameter of the Universe. It is found that due to the limitation of the present observation value of the equation of state of dark energy, the non-gravitational interaction function takes a positive value, the energy is transferred from cold dark matter to dark energy, and there is a simple inverse linear relationship between the present value of equation of state and the interaction factor. The cosmic deceleration parameter $q$ is negative, which is consistent with the cosmic acceleration expansion observed in astronomy.

## II Ghost dark energy model

Recently, references [41–48] have proposed a new dark energy model, the so-called Veneziano ghost dark energy. The key element of this new model is that the Veneziano ghost is unphysical in the usual Minkowski spacetime quantum field theory, but it exhibits important physical effects in dynamical spacetime or spacetime with nontrivial topology. Veneziano ghost is supposed to exist for solving the U(1) problem in low energy effective theory of QCD [49–52]. In Minkowski spacetime the ghost has no contribution to the vacuum energy density. However, in a curved spacetime it gives rise to a small vacuum energy density proportional to $\Lambda^3_{QCD}H$ [53–55], where $\Lambda_{QCD}$ is QCD mass scale with $\Lambda_{QCD} \sim 100$MeV and $H \sim 10^{-33}$eV is the Hubble parameter. It is important to note that $\Lambda^3_{QCD}H \sim (3 \times 10^{-3}\text{eV})^4$ gives the right order of observed dark energy density. This numerical coincidence is conspicuous and also means that this model gets rid of the fine tuning problem [41,42,44]. Also, note that in this ghost dark energy model, there are no unwanted features, such as violation of gauge invariance, unitarity, causality, etc. [41, 43, 53–55]. In fact, as the Veneziano ghost is not a physical propagating degree of freedom, the description in terms of the Veneziano ghost is just a matter of convenience to describe very complicated infrared dynamics of strongly coupled QCD. Without using the ghost field one can describe the same dynamics by some other approaches (e.g. direct lattice simulations). The biggest advantage of this model is that it is totally embedded in both standard model and general relativity: one needs not introduce any new parameter, new degree of freedom, or modify gravity. To enable the equation of state for dark energy to cross -1, Veneziano ghost fields are introduced, which are distinct from those in some dark energy models in other literature, where the ghost fields are real physical degrees of freedom. On the other hand, the vacuum energy is generally expected to be exponentially suppressed because QCD is a theory with a mass gap, rather than being linear as in the Hubble parameter. This interesting proposal claimed that the cosmological constant arises from the contribution of the ghost fields which are supposed to be present in the low-energy effective theory of QCD. Its main point is that the dark energy density is proportional to the Hubble parameter, i.e.,$\rho_D = \alpha H$. Although reference [47] thinks that the dark energy density includes the quadratic power of $H$, it is simplest to only include the first power of $H$, which allows the model we study to provide an analytical solution.

## III Interacting ghost dark energy model

The interacting ghost dark energy model assumes that there is a non-gravitational interaction between cold dark matter and dark energy. Nevertheless, it assumes that dark energy comes from the contribution of a ghost field. Since the WMAP 5-year data [56] constrains the curvature of the present Universe to be $-0.0075 < \Omega_K < 0.0085$ at the 95% confidence level, and the ghost field dark energy is a part of cosmic matter, we can generalize the flat universe to a non-flat Universe.

The line-element describing a 4-dimensional homogeneous and isotropic Universe, which is called the FLRW space-time, is given by [57]

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{1}$$

where $a$ is the scale factor with cosmic time $t$, and $K$=-1, 1, 0 corresponds to open, closed and flat geometries, respectively. The cosmological dynamics is known by solving the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2}$$

For the metric (1), the (00) component of the Einstein equations gives [57]

$$H^2 + \frac{K}{a^2} = 8\pi G(\rho_m + \rho_D) \tag{3}$$

where $\rho_m$ and $\rho_D$ are zero pressure matter energy density and ghost field dark energy density, respectively. The dimensionless energy density is defined as

$$\Omega_D = \frac{8\pi G\rho_D}{H^2}, \Omega_m = \frac{8\pi G\rho_m}{H^2}, \Omega_K = \frac{-K}{a^2H^2} \tag{4}$$

The Friedmann Eq. 3 becomes

$$\Omega_D + \Omega_m + \Omega_K = 1 \tag{5}$$

The conservation equations of cold dark matter without pressure and dark energy are assumed to be

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{6}$$

and

$$\dot{\rho}_D + 3H\rho_D(1+\omega_D) = -Q \tag{7}$$

respectively (which corresponds to the conservation of the energy-momentum tensor for each component after introduction of $Q$ term), where $\omega_D$ is the ratio of dark energy pressure to its density, that is, the parameter of the equation of state of dark energy and $Q$ represents the non-gravitational interaction term, which can be in different forms. We take non-gravitational interaction function as $Q$=$3H\delta\rho_m$ with an interaction factor $\delta$. It is worth noting that the continuity equations imply that the non-gravitational interaction function should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor) multiplied by the energy density. Therefore, the interaction term could be in different forms; for example, the reference [45, 48] had considered $Q\propto$ H$(\rho_m + \rho_D)$. Our selection form of $Q$ is the simplest, and the physical meaning is clearer. It can be obtained from Eq. 6 that $\rho_m = \rho_{m0}a^{-3(1-\delta)}$. In order for the density of cold dark matter to decrease with the increase of $a$, the interaction factor $\delta$ must be less than 1. However, the observational present-day value of the equation of state of dark energy $\omega_{D0}$ has still more strict restrictions on the interaction factor that we will point out below. Using the dimensionless quantity in (4), (6) reads

$$\dot{\Omega}_m + 2\frac{\dot{H}}{H}\Omega_m + 3H\Omega_m = Q \tag{8}$$

For the same reason, the Eq. 7 results in

$$\dot{\Omega}_D + 2\frac{\dot{H}}{H}\Omega_D + 3(1+\omega_D)H\Omega_D = -Q \tag{9}$$

Eq. 5, Eq. 8 and Eq. 9 lead to

$$\omega_D = -\frac{1}{\Omega_D}(\frac{2}{3}\frac{\dot{H}}{H^2} + 1 - \frac{1}{3}\Omega_K) \tag{10}$$

Inserting $\dot{\rho}_m$ from Eq. 3 into Eq. 6, and considering $\rho_D = \alpha H$, we get

$$\frac{\dot{H}}{H^2} = -\frac{3\Omega_m(1-\delta) + 2\Omega_K}{(2-\Omega_D)} \tag{11}$$

Then

$$\omega_D(a,\delta) = -\frac{1}{2-\Omega_D}[1 + \frac{1}{3}\Omega_K + 2\delta\frac{\Omega_m}{\Omega_D}] \tag{12}$$

and $\Omega_D = \Omega_{D0}H_0/H$ with $a = a_0 = 1, H = H_0, \Omega_D = \Omega_{D0}$.

From Eq. 3, we can get

$$E = \frac{H}{H_0} = \frac{1}{2}\Omega_{D0} + \sqrt{\frac{1}{4}\Omega_{D0}^2 + \Omega_{K0}a^{-2} + \Omega_{m0}a^{-3(1-\delta)}} \tag{13}$$

It should be noted that there are two subtractions in Eq. 13, one of which is that $E(a=1)=0$ corresponding to the last shrinking universe, which has been omitted. Therefore

$$\Omega_D = \Omega_{D0}E^{-1}, \Omega_m = \Omega_{m0}E^{-2}a^{-3(1-\delta)}, \\ \Omega_K = \Omega_{K0}a^{-2}E^{-2} \tag{14}$$

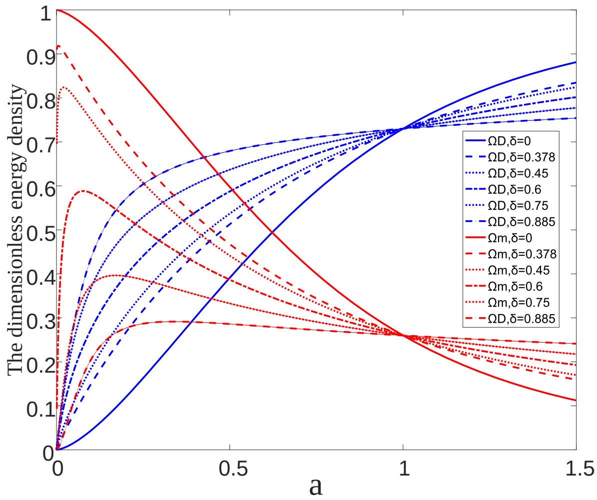


Figure 1: The dimensionless energy density vs. a.

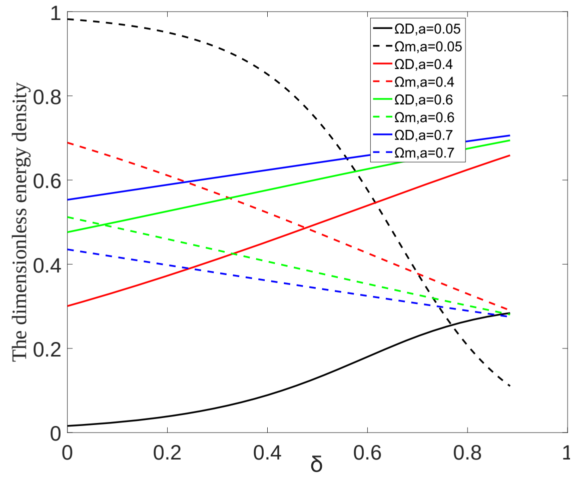


Figure 2: The dimensionless energy density vs. $\delta$.

These energy densities are in analytic form. Fig. 1 shows the change of dimensionless energy density with cosmic scale factor $a$, and Fig. 2 shows the change of dimensionless energy density with interaction factor $\delta$ in which $\Omega_{m0}$=0.26, $\Omega_{D0}$=0.73 and $\Omega_{K0}$=0.01. The line in Fig. 1 with the interaction factor $\delta$=0 corresponds to non-interaction of cold dark matter to dark energy. The image shows that with the increase of the cosmic scale factor, in the absence of non-gravitational interaction, the dimensionless energy density of cold dark matter gradually decreases from the initial maximum, while the dimensionless dark energy density gradually increases from the minimum owing to the contribution of the ghost field. With the emergence of non-gravitational interaction, the density of cold dark matter immediately decreases; that is, it is transformed into dark energy. Of course, when the interaction factor $a$ is in the range of 0.45 to 0.885, the dark matter energy density will initially increase in a small process, but will soon decrease, that is, into dark energy. This is because the astronomical observations of $\Omega_{D0}$ require that the interaction factor $\delta$ be positive, resulting in a positive non-gravitational function $Q$. Unlike in [58], where $Q = 3\delta H(1+\omega_D)\rho_D$, for the regime $\omega_D > -1$ the energy flow takes place from cold dark matter to dark energy, while for the $\omega_D < -1$ regime the energy transformation reverses its direction, and it is believed that both situations may occur.

When the cosmic scale factor $a$ is constant in Fig. 2, the non-gravitational interaction between dark energy and cold dark matter makes $\Omega_m$ decrease and $\Omega_D$ increase; that is, zero pressure cold dark matter is transformed into dark energy. If the present dimensionless curvature $\Omega_{K0}$ is closed to 0.01($K$=-1), then the universe is open. It can be seen from Eq. 14 that the dimensionless curvature $\Omega_k$ decreases gradually.

The bound on the present equation state of dark energy from the WMAP 5-year data combined with other observational data is $-1.29 < \omega_{D0} < -0.790$

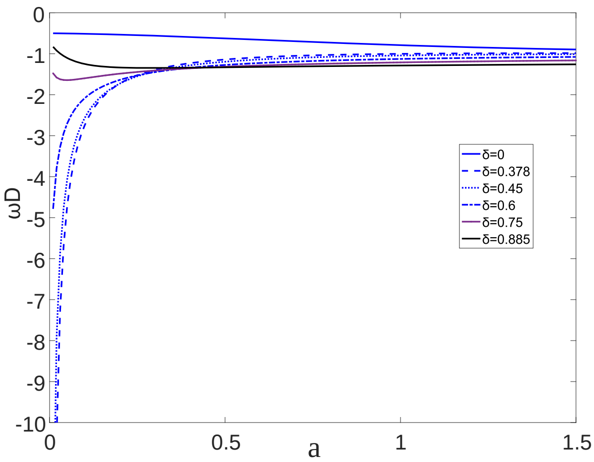


Figure 3: The evolution of the equation of state with different values of $\delta$.

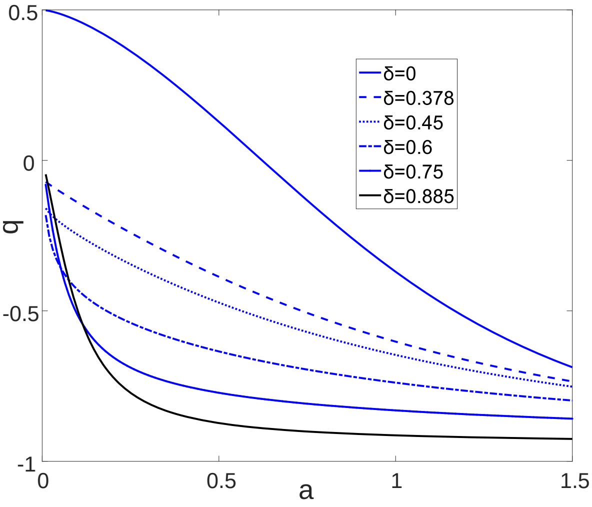


Figure 4: The deceleration parameter $q$ with different values of $\delta$.

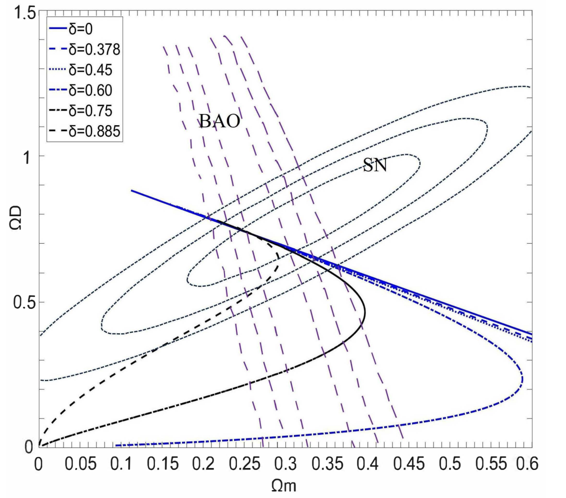


Figure 5: Contour plots of $\Omega_D$ vs. $\Omega_m$. SN and BAO regions are adapted from reference [60].

at the 95% confidence level [56]. It can be seen from Eq. 12 that there is a simple inverse linear relationship between $\omega_{D0}$ and factor $\delta$. The range of present value of the equation of state $\omega_{D0}$ limits the range of $\delta \in (0.0048, 0.894)$. The evolution of the equation of state with different values of $\delta$ is shown in Fig. 3. The line with the interaction factor $\delta$=0.378 corresponds to the equation of state $\omega_{D0}$=-1. Both the values $\omega_{D0}$=-0.784 with $\delta$=0, $\Omega_{K0}$=0, and $\omega_{D0}$=-0.787 with $\delta$=0, $\Omega_{K0}$=0.01 are clearly out of the range. Therefore, we can conclude that the universe is flat and the interaction between dark energy and cold dark matter does not exist, which is probably not allowed. Of course, because the observed value of $\Omega_K(-0.0175 < \Omega_{K0} < 0.0085)$ is very small, we can consider that the universe is approximately flat, but there must be interaction between dark energy and cold dark matter, therefore we take the value of $\Omega_{K0}$ to be 0.01.

We can also calculate the deceleration parameter, which is defined as

$$q = -1 - \frac{\dot{H}}{H^2} \tag{15}$$

After inserting Eq. 11 into Eq. 15, we get

$$q = -1 + \frac{3\Omega_m(1-\delta) + 2\Omega_K}{2 - \Omega_D} \tag{16}$$

The expression of energy density $\Omega_m, \Omega_D$ and $\Omega_K$ has been given in Eq. 14. The deceleration parameter $q$ relative to the cosmic scale factor is shown in Fig. 4. We can see in the picture that all values of the deceleration parameters $q_0 \in [-0.91, -0.38]$ are negative, which is in line with the astronomical observation of the accelerated expansion of the universe. Our results are consistent with one in reference [59] in which $q_0 = -0.48 \pm 0.11$ from samples of 192 supernovae and $q_0 = -0.65 \pm 0.5$ from the radio galaxies. In addition, we also observe that the deceleration parameter $q$ is related to the interaction factor $\delta$. The stronger the interaction, the more negative is the deceleration parameter, and the faster the accelerated expansion.

Fig. 5 shows the contour plots of the dimensionless dark energy density $\Omega_D$ relative to the density of cold dark matter $\Omega_m$ with SN and BAO regions adapted from reference [60] for comparison. As can be seen from the figure, when the interaction factor $\delta$ is 0.60, 0.75 and 0.885, the density of dimensionless cold dark matter has a process of increasing, but will also decrease after reaching the maximum value, while the density of dimensionless dark matter always increases. $\Omega_D$ and $\Omega_m$ starting from zero seem to be more consistent with the principle that the universe came from nothing.

In order to show the role of dark energy in accelerating the expansion of the universe, we plot

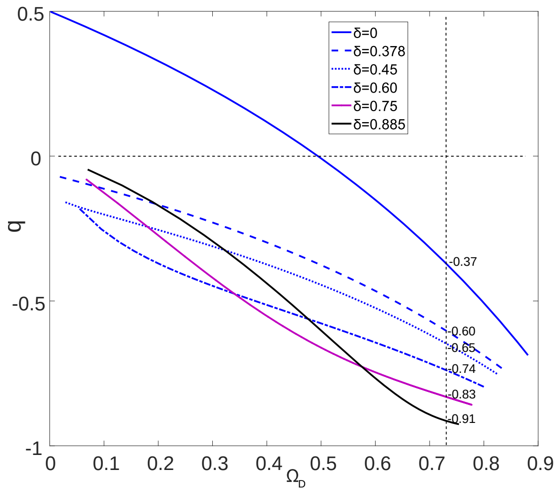


Figure 6: The deceleration parameter q vs. $\Omega_D$

the cosmic deceleration parameter with the dimensionless dark energy density, as shown in Fig. 6, where the values of the deceleration parameter corresponding to the values of different interaction factors have been marked at cosmic scale factor $a$=1.

## IV Conclusions

In this note the interacting ghost dark energy model is solved in which the non-gravitational interaction function between cold dark matter and dark energy can be introduced under the condition that the total energy density is conserved. The advantage of this model is that the dimensionless energy density, equation of state parameters and cosmic deceleration factor can be given analytically. In the original interaction dark energy model, the non-gravitational interaction function can be positive or negative, and the energy conversion can be from cold dark matter to dark energy, or reverse flow. It is found that, in the model given in this note, the non-gravitational interaction function between cold dark matter and dark energy prefers the positive value, which results in energy transfer from cold dark matter to dark energy. The value of the interaction factor is selected with the bound on the present equation state of dark energy, and the present value of the deceleration factor given by the value of the factor is in good agreement with the observed value.

*Acknowledgements* - This report received funding from the Guangxi University Science and Technology Research Project (KY2015LX329).

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