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## Commentary on “Effect of temperature on a granular pile”

Antonio Coniglio<sup>1\*</sup>, Massimo Pica Ciamarra<sup>2</sup>

The author of [1] writes an interesting and critical review on the effect of temperature in a granular pile. First, it is shown the effect of temperature in several experiments. Then, it is shown how the compaction experiments, originally done by shaking granular material, can equally be performed by thermal cycling.

To a first approximation, one may expect that at the particle level the only effect of a temperature variation is a small particle volume change. If this is the case, in a thermal cycle the volume fraction of the system changes, and a thermal cycle could be seen as a compression cycle. This analogy suggests that the initial and final states of a cycle differ because of the disorder of the system. To understand this point, it is convenient to first consider the effect of a particle expansion cycle in a crystalline structure of spherical balls. In this structure, all inter-particle forces are equal in magnitude, and the net force acting on each particle is zero. On inflating the particles, all forces increase exactly by the same amount, and the net force acting on each particle never changes. Accordingly, particles keep their position and no motion occurs. Conversely, in the presence of disorder, the inter-particle forces are different and vary by different amounts on inflating the particles. Particle swelling drives the

system out of mechanical equilibrium, and induces motion. Using different words, one may say that ordered structures respond in an affine way to particle swelling, while disordered structures are characterized by a non-affine response.

This picture, which is also valid for frictionless particles, becomes more involved in the presence of friction. In fact, one should consider that the microscopic origin of friction is in the asperities of the surfaces; the shearing of the frictional contacts induced by the thermal cycle, may cause them to break. In a series of thermal cycles, contacts repeatedly break, allowing the system to compact. One may also speculate that, apart from the temperature variation which controls the relative volume change of the particles, an important control parameter in thermal cycles is the absolute value of the temperature. In fact, the height of the asperities is expected to decrease as particles become bigger.

The role of friction in granular materials has been extensively investigated in the literature (see, for instance, the paper by Song *et al.* [5] and a recent review [4]), and we have recently proposed a jamming phase diagram in a three dimensional space, where the axes are volume fraction, shear stress and friction coefficient [2]. In this line of research, the results of Ref. [1] are of particular interest; in fact, relating friction to temperature may allow to experimentally tune the friction coefficient and to validate different proposed theoretical scenarios.

We take the occasion to present a speculative picture regarding the role of friction in sheared granular systems, making an analogy between frictional

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\*E-mail: coniglio@na.infn.it

<sup>1</sup> Dip.to di Scienze Fisiche, Università di Napoli “Federico II” and CNR-SPIN, Naples, Italy

<sup>2</sup> CNR-SPIN, Università di Napoli “Federico II”, Monte S. Angelo Via Cinthia, 80126 Napoli, Italy.

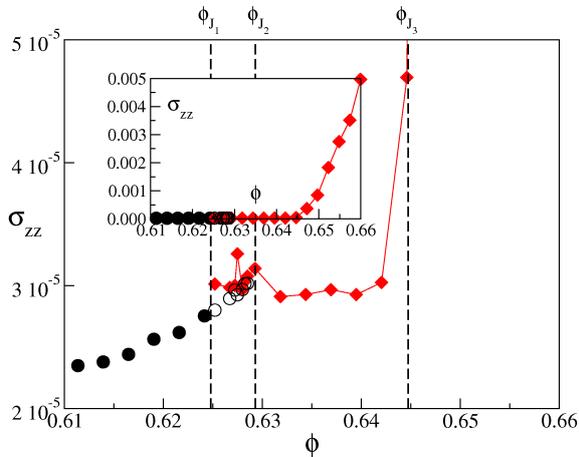


Figure 1: Pressure  $\sigma_{zz}$  as a function of the volume fraction  $\phi$ , for  $\sigma = 2 \cdot 10^{-3}$ , and  $\mu = 0.1$  in a small (main panel) and in a much larger (inset) pressure range. Circles correspond to measures taken when the system flows, and diamonds to measures taken in the jammed phase. Full symbols correspond to measures taken in the steady state, while the open circles for  $\phi_{J_1} < \phi < \phi_{J_2}$  correspond to measures taken in flowing metastable states which jam at long times.

sheared granular and thermal systems. In this picture, we speculate an analogy between the ratio  $\mu/\sigma$  and the ratio  $\varepsilon/T$ . Here,  $\mu$  and  $\sigma$  are the friction coefficient and the shear stress of a granular system, while  $\varepsilon$  measures the strength of the attractive force between thermal particles, and  $T$  is the temperature. We associate  $\mu$  to  $\varepsilon$ , as higher the  $\mu$ , stickier the contact, i.e. the greater the shear force the contact is able to sustain.

We have investigated the limit of validity of this analogy performing molecular dynamics simulations of sheared granular systems in three dimensions. Particles are confined between two parallel rough plates in the  $x$ - $y$  plane, the bottom plate is fixed, while the other may move. We apply to them a shear force. Periodic boundary conditions are along  $x$  and  $y$ . Details of the numerical model and of the investigated system are in Ref. [2,3]. We vary the volume fraction (changing the number of particles at constant volume), the shear stress  $\sigma_{xy}$  and the friction coefficient  $\mu$ .

Figure 1 illustrates a typical pressure versus vol-

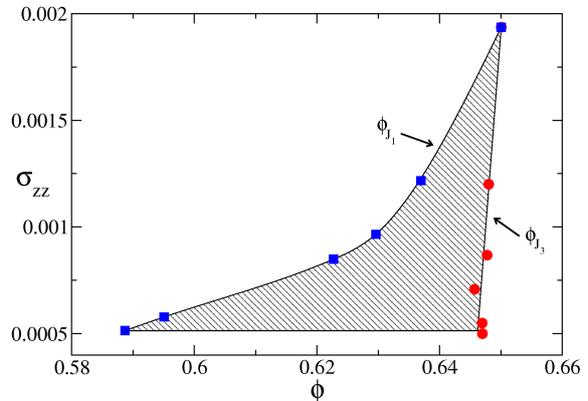


Figure 2: Normal pressure of the confining plate  $\sigma_{zz}$  as a function of the volume fraction  $\phi$  at  $\sigma = 5 \cdot 10^{-2}$ . Different symbols correspond to  $\phi_{J_1}$  (squares) and  $\phi_{J_3}$  (circles), for different values of the friction coefficient, from  $\mu = 0$  (top) to  $\mu = 0.8$  (bottom). The shaded area can therefore be identified with the coexistence region.

ume fraction curve, for a fixed value of the shear stress, where three transitions are enlighten. Here,  $\sigma_{zz}$  is the normal force acting on the confining plate per unit surface. For  $\phi < \phi_{J_1}$ , the system is in a steady flowing state. For  $\phi_{J_1} < \phi < \phi_{J_2}$ , the system is found either in a metastable flowing state, or in an equilibrium disordered solid state able to sustain the applied stress. When in the metastable state, the system flows with a constant velocity for a long time, but it suddenly jams in an equilibrium solid-like state.<sup>1</sup> For  $\phi_{J_2} < \phi < \phi_{J_3}$ , the system quickly jams in response to the applied stress. For  $\phi > \phi_{J_3}$ , the system responds as a solid to the applied stress. The equilibrium value of the pressure  $\sigma_{zz}$ , marked by solid symbols in Fig. 1, increases in the flowing state, it discontinuously jumps to a different value at  $\phi_{J_1}$ , and grows for  $\phi > \phi_{J_3}$ . This scenario, and particularly the presence of a density range where the pressure is constant, suggests to interpret the  $\phi_{J_1}$ - $\phi_{J_3}$  segment as a coexistent line.

We have investigated the limit of validity of this scenario performing a number of simulations at different values of the friction coefficient  $\mu$ . In the proposed analogy, low values of  $\mu$  correspond to a high

<sup>1</sup>The terms ‘metastable’ and ‘equilibrium’ are used to indicate states with a finite/infinite lifetime, respectively.

$T/\varepsilon$  ratio. For each value of  $\mu$ , we have estimated  $\phi_{J_1}(\mu)$  and  $\phi_{J_3}(\mu)$ , which are the two extrema of the coexistence line at that value of  $\mu$ . As shown in Fig. 2, these two lines allow to identify the (analogous to the) region in the  $\sigma_{zz}$ - $\phi$  plane. At  $\mu = 0$ ,  $\phi_{J_1}(\mu) = \phi_{J_3}(\mu)$ , and the coexistence area ends in what should be the critical point, which here occurs at infinite temperature (as  $\mu \propto 1/T = 0$ ). At finite friction,  $\phi_{J_1}(\mu) < \phi_{J_3}(\mu)$ , and coexistence lines are found, as shown in the figure for few values of  $\mu$ . The coexistence area has a lower bound, which is found in the limit of high friction.

These results suggest that it is not unreasonable to associate the friction coefficient of sheared granular systems to the inverse temperature of thermal systems. A deeper investigation is required to define the limits of validity of this analogy.

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