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# A novel singularity-free black hole with nonlinear magnetic monopole: Hawking radiation and quantum correction

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This paper introduces a nonlinear, magnetically charged, singularity-free black hole model. The Ricci scalar, Kretschmann scalar, horizon, energy conditions, and Hawking radiation corresponding to the singularity-free metric are presented, and the asymptotic behavior and quantum correction of the model are examined. The model was constructed by coupling a mass function with the regular black hole solution under nonlinear electrodynamics in general relativity. Aside from resolving the problem of singularities in Einstein's theory of general relativity, the model asymptotically meets the quantum correction under an effective field theory. This obviates the need for additional correction terms; in this regard, the model outperforms the black hole models developed by Bardeen and Hayward. Regarding the nonlinear magnetic monopole source of the gravitational field of the black hole, the energy-momentum tensors fulfill weak energy conditions. The model constitutes a novel, spherically symmetric solution to regular black holes.

## I Introduction

Newton's law of gravitation states that the collapse of a nonrotating, perfectly spherical dust shell leads to a singularity at the center because all of the matter simultaneously reaches  $r = 0$ . Subsequently, singularities would not occur if the initial configuration of the matter were slightly distorted [1]. Therefore, Huang (2020) [2] proposed a modification to Newton's gravitation which obeys the inverse square law and does not have singularities at  $r = 0$ . According to general relativity, the presence of trapped

surfaces is the key to the formation of singularities from gravitational collapse. They are surfaces on which the radial coordinates of particles following a timelike or a null curve can gradually only go to reducing values. These trapped surfaces are subject to an extreme gravitational field, where light emitted from the surface is dragged backward, and describe the inner region of an event horizon. This theoretical singularity exists in static black holes [3, 4]. It follows the singularity theorem proposed by Penrose and Hawking [5–7].

However, it is possible to speculate the existence of singularity-free (regular) black holes. When curvature increases (i.e., when the Planckian value is reached), general relativity should be modified to resolve singularity. Accordingly, Bardeen (1968) [8] proposed the first static spherically symmetric regular black hole solution. This was followed by Dymnikova (1992) [9]; Mazur and Mottola (2001) [10]; Nicolini (2005) [11]; Hayward (2006) [12]; Hossenfelder, Modesto, and Pemont-Schwarz (2010) [13]. These above-mentioned regular black hole solutions

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all satisfy the weak energy conditions. Among them, the Hayward metric is the simplest model. In addition, the regular black hole model established by E Ayón-Beato and A García (1998) [14–16] can be interpreted as a nonlinear electric or magnetic gravitational field monopole. In the past two decades, some interesting solutions to Einstein’s equations of general relativity have been constructed under the framework of nonlinear electrodynamics (NED) [17–22]. A recent study showed that in the Bardeen model, parameter  $g$  is a magnetic monopole gravitational field described by NED [23]. However, the electromagnetic tensors used in Bardeen’s solution are stronger than those used in Maxwell electrodynamics when the limit of weak magnetic fields is calculated [1]. To address this issue, Kruglov (2017) [24] derived a magnetic black hole solution from the exponential nonlinear framework of electrodynamics (ENE).

Literature asserted that metrics that can successfully simulate quantum effects must meet the “one-loop quantum correction” of Newtonian potential, obtained through effective field theory [25–28]. The Bardeen model meets the quantum correction asymptotically [27]; however, the Hayward model does not, requiring additional correction terms to be introduced [29].

This study aims to propose a novel singularity-free black hole model, unlike Bardeen, Hayward, and Kruglov’s ENE models. The proposed model, based on the recent modification of Newtonian gravity by Huang (2020) [2], is an extended version in the framework of general relativity. It resolves the singularity problem in Einstein’s theory, satisfies weak energy conditions, and returns the electromagnetic field tensors of the Lagrange function for the nonlinear magnetic monopole gravitational field to Maxwell electrodynamics when calculating the weak magnetic field limit. Simultaneously, the metric meets the quantum correction asymptotically, without additional correction terms.

This paper is organized as follows: First, we discuss the energy–momentum tensors of nonlinear electrodynamics, going on to present a nonlinear, magnetically charged, and singularity-free black hole model developed by introducing mass functions to the metric. Second, we present the Ricci scalar, the Kretschmann’s scalar, the horizon, the energy conditions, and the Hawking radiation for the metric. Third, we discuss the asymptotic behavior and

quantum correction. We set the following parameter:  $c = G = 1$ . The first and second partial differential of  $f(r)$  on  $r$  are noted as  $f'$  and  $f''$ .

## II Energy–momentum tensor of nonlinear electrodynamics in general relativity

In this section, we propose the energy–momentum tensor of nonlinear electrodynamics under the general relativity framework, a method used first by E Ayón-Beato and A García [23]. Consider the following action that represents nonlinear electrodynamics in curved spacetime:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - \mathcal{L}(\mathcal{F})), \quad (1)$$

where  $\mathcal{F} = F_{ab}F^{ab}$  is the square of the electromagnetic field strength tensor,  $\mathcal{L}$  is a Lagrangian density function associated with  $\mathcal{F}$ , and  $\mathcal{L}_{\mathcal{F}} = \frac{\partial \mathcal{L}}{\partial \mathcal{F}}$ . The electromagnetic tensor  $F_{ab}$  is defined based on the vector potential  $A_a$ :

$$F_{ab} = \nabla_a A_b - \nabla_b A_a. \quad (2)$$

Einstein’s equations are derived using Eq. (1):

$$G_{ab} = T_{ab}, \quad (3)$$

$$T_{ab} = 2 \left( \mathcal{L}_{\mathcal{F}} F_{ab}^2 - \frac{1}{4} g_{ab} \mathcal{L} \right). \quad (4)$$

The nonlinear Maxwell equations can be expressed as follows:

$$\nabla_a (\mathcal{L}_{\mathcal{F}} F^{ab}) = 0. \quad (5)$$

When  $\mathcal{L}(\mathcal{F}) = \mathcal{F}$ , Eq. (5) regresses to the standard Maxwell equations.

We start with a following generalized spherical symmetry metric:

$$\begin{aligned} ds^2 &= -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \\ d\Omega^2 &= d\theta^2 + \sin^2 \theta d\phi^2, \\ A &= a(r)dt + Q_m \cos \theta d\phi, \end{aligned} \quad (6)$$

where  $Q_m$  represents the overall magnetic charge and can be defined as follows:

$$Q_m = \frac{1}{4\pi} \int \mathcal{F}. \quad (7)$$

We found it extremely difficult to construct a solution to analyze black holes with hadronic charges. However, we found it significantly easier to construct a solution for single charges, i.e.,  $a(r) = 0$  or  $Q_m = 0$ . Therefore, we explicitly explain the use of the abovementioned magnetic charge to construct an exact black hole solution.

The primary motivation of this study is to create a singularity-free black hole using the proposed gravitational model. Therefore, we initially focused on identifying the metrics that were consistently instructive at the origin of spacetime. We parameterized the metric function as follows:

$$f(r) \equiv 1 - \frac{2M(r)}{r}. \quad (8)$$

The constant mass of Schwarzschild's black hole was replaced with a mass distribution function  $M(r)$ .

Using the magnetically charged exact black hole solution as an example, the metric function can be parameterized using Eq. (6) and by setting  $a(r) = 0$ . The results indicate that the nonlinear Maxwell equations are self-satisfying.

We find only two following independent equations for Einstein's equations:

$$0 = \frac{f'}{r} + \frac{f-1}{r^2} + \frac{1}{2}\mathcal{L}, \quad (9)$$

$$0 = f'' + \frac{2f'}{r} + \mathcal{L} - \frac{4Q_m^4}{r^4}\mathcal{L}_{\mathcal{F}}. \quad (10)$$

The Lagrangian density  $\mathcal{L}$  can be solved to be served as a function for  $r$ .

$$\mathcal{L} = -2 \left( \frac{f'}{r} + \frac{f-1}{r^2} \right). \quad (11)$$

Eq. (11) was incorporated into Eq. (10). We found that for any given metric function, the latter function is always self-satisfying. Therefore, the most common method is to use Eq. (6) to derive magnetically charged spherically symmetric static solutions. During the parameterization of Eq. (8), the Lagrangian density is simplified as follows:

$$\mathcal{L} = \frac{4M'(r)}{r^2}. \quad (12)$$

In addition,  $\mathcal{F}$  can be expressed as follows:

$$\mathcal{F} = F_{ab}F^{ab} = \frac{2Q_m^2}{r^4}. \quad (13)$$

Therefore, one can freely select a mass function  $M(r)$  of interest and then analytically resolve the Lagrangian density to use it as a function for  $\mathcal{F}$ . This completes the calculations for a magnetically charged static solution.

### III Construction of a singularity-free black hole model coupled with nonlinear electrodynamics

In this section, we construct a singularity-free Newtonian theory of gravity under the framework of general relativity in curved spacetime using the results of the previous section. For this purpose, we incorporated a mass function  $M(r)$  to couple with the regular black hole solution with nonlinear electrodynamics in general relativity. The proposed model is as follows:

$$M(r) \equiv M \left( \frac{r}{r+h} \right)^\mu, \quad (14)$$

$$f_{\text{CH}}(r) \equiv 1 - \frac{2M(r)}{r},$$

$$ds^2 = -f_{\text{CH}}(r)dt^2 + f_{\text{CH}}(r)^{-1}dr^2 + r^2d\Omega^2,$$

where a small constant  $h$  (unit: length) was inserted to prevent divergence of the equation when  $r \rightarrow 0$ .  $f_{\text{CH}}(r)$  is referred to as the Chou-Huang function, and  $\mu$  is a dimensionless parameter that should be solved to satisfy the Einstein–Maxwell equations,  $\mu > 0$ , while  $M$  is a constant denoting gravitational mass. When  $\mu = 1$  and  $h \rightarrow 0$ , the metric regresses to the Schwarzschild's metric with a mass of  $M$ . This prevents the scalar curvature from diverging when  $r \rightarrow 0$ . The Ricci scalar is expressed as follows:

$$R = \frac{2\mu(\mu+1)Mh^2}{r^{(3-\mu)}(r+h)^{\mu+2}}. \quad (15)$$

This equation highlights that  $R$  satisfies the condition of nondivergence for the scalar curvature when  $\mu \geq 3$ . To simplify calculations, we only discuss  $\mu = 3$  in this study. By inserting Eq. (15), we obtain the following:

$$R = \frac{24Mh^2}{(r+h)^5}. \quad (16)$$

Furthermore, we prove the metric couples to Maxwell electrodynamics in the weak-field limit. Einstein's equations (9)-(10) and Eq. (12) with Lagrangian density are solved regarding mass function  $M(r)$ , obtaining the mass function in the following form:

$$M(r) = M - \beta \frac{h^3}{\alpha} \left[ 1 - \left( \frac{r}{r+h} \right)^3 \right], \quad (17)$$

where  $\alpha$  is a constant (unit: length squared), and satisfies  $\alpha = \frac{h^4}{2Q_m^2}$ . The term  $\beta \frac{h^3}{\alpha}$  denotes the electromagnetic-induced mass  $M_{em}$ , and  $\beta$  denotes a dimensionless constant. We assumed the gravitational mass  $M$  to be equal to  $M_{em}$ , i.e.,  $M = M_{em} \equiv \beta \frac{h^3}{\alpha}$ . Subsequently, Eq. (17) would return to the form of Eq. (14). Using the definition of  $\mathcal{F}$  in Eq. (13), we obtained the following:

$$\alpha \mathcal{F} = \left( \frac{h^4}{2Q_m^2} \right) \left( \frac{2Q_m^2}{r^4} \right) = \frac{h^4}{r^4}. \quad (18)$$

We simplified the Lagrangian density to the following:

$$\mathcal{L} = \frac{4M'(r)}{r^2} = \frac{12\beta}{\alpha} \left( \frac{h}{r+h} \right)^4 = \frac{12\beta \mathcal{F}}{(1 + (\alpha \mathcal{F})^{1/4})^4}. \quad (19)$$

Thereafter, we used  $M$  and  $Q_m$  to express  $\alpha$ .

$$\mathcal{L}(\mathcal{F}) = \frac{12\beta \mathcal{F}}{\left( 1 + \left( \frac{8Q_m^6}{M^4} \mathcal{F} \right)^{1/4} \right)^4}, \quad (20)$$

where the Lagrangian contains fractional powers of  $\mathcal{F}$  and  $\mathcal{F} = F_{ab}F^{ab} = \frac{2Q_m^2}{r^4} \geq 0$ . Under the weak magnetic field limit,  $M \gg Q_m$  and  $\mathcal{L}(\mathcal{F}) \rightarrow 12\beta \mathcal{F}$ . Therefore, when  $\beta = 1/12$ ,  $\mathcal{L}(\mathcal{F})$  regresses to Maxwell electrodynamics. Finally, we solved the Chou-Huang function and obtained the following:

$$f_{CH}(r) = 1 - \frac{2M}{r} \left( \frac{r}{r+h} \right)^3. \quad (21)$$

The calculations were inserted into Eq. (14) to obtain a singularity-free metric. The metric line elements are the following:

$$ds^2 = - \left( 1 - \frac{2Mr^2}{(r+h)^3} \right) dt^2 + \left( 1 - \frac{2Mr^2}{(r+h)^3} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (22)$$

The Ricci scalar and Kretschmann's scalar are expressed as follows:

$$R = \frac{24Mh^2}{(r+h)^5}, \quad (23)$$

$$K = \frac{48M^2(2h^4 + 7h^2r^2 - 2hr^3 + r^4)}{(r+h)^{10}}. \quad (24)$$

When  $h \rightarrow 0$ , the metric is restored to Schwarzschild's metric with a constant mass of  $M$ . The asymptotic of the Ricci and Kretschmann scalars can be obtained as follows:

$$R = \frac{24M}{h^3} - \frac{120Mr}{h^4} + \frac{360Mr^2}{h^5} + O(r^3) \quad r \rightarrow 0, \quad (25)$$

$$R = \frac{24Mh^2}{r^5} - \frac{120Mh^3}{r^6} + O(r^{-7}) \quad r \rightarrow \infty, \quad (26)$$

$$K = \frac{96M^2}{h^6} - \frac{960M^2r}{h^7} + \frac{5616M^2r^2}{h^8} + O(r^3) \quad r \rightarrow 0, \quad (27)$$

$$K = \frac{48M^2}{r^6} - \frac{576M^2h}{r^7} + O(r^{-8}) \quad r \rightarrow \infty. \quad (28)$$

The Ricci and Kretschmann scalars vanish, and the spacetime becomes flat when  $r$  approaches infinity. Eqs (25)–(28) indicate that the metric in Eq. (22) is regular. We thus complete the extension of revising Newtonian gravity under the general relativity framework.

## IV Energy condition

We note that Eq. (20) satisfies weak energy conditions. Let  $X$  be a timelike field without loss of generality.  $X$  can be selected as a normal field (i.e.,  $X_a X^a = -1$ ). The local energy density along  $X$  can be expressed using the right side of Eq. (4), as follows:

$$T_{ab} X^a X^b = 2 \left( E_\gamma E^\gamma \mathcal{L}_{\mathcal{F}} + \frac{1}{4} \mathcal{L} \right). \quad (29)$$

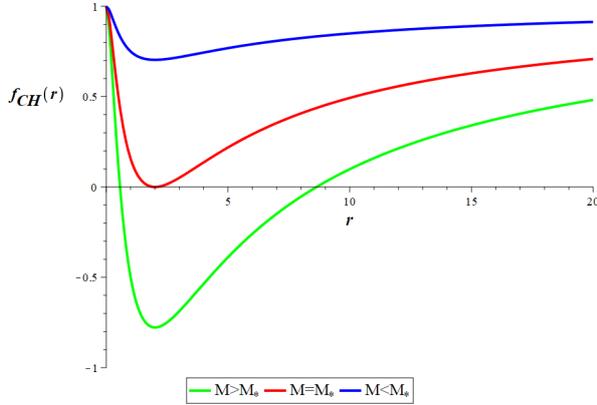


Figure 1: Plot of the Chou-Huang function  $f_{\text{CH}}(r)$ . Variation of the  $f_{\text{CH}}(r)$  with different values of  $M_c$  while maintaining  $h = 1$ , when  $r > 0$ ;  $f_{\text{CH}}(r) = 0$  is the future trapping region:  $M > M_*$  contains two horizons,  $M = M_*$  contains one horizon, and  $M < M_*$  does not contain any horizons.

By definition,  $E_\gamma = F_{\gamma\delta}X^\delta$  is orthogonal to  $X$ . Therefore, it is a spacelike vector ( $E_\gamma E^\gamma > 0$ ). Using Eq. (29), we could determine that if  $\mathcal{L} \geq 0$  and  $\mathcal{L}_{\mathcal{F}} \geq 0$ , there would not be any negative local energy densities anywhere in the field. This is a weak energy condition. The quantities of nonnegativity were derived using Eq. (20). Therefore, the proposed model satisfies weak energy conditions.

## V Horizon

$g_{tt} = 0$  was used to infer the horizon of the black hole.

$$f_{\text{CH}}(r) = 1 - \frac{2Mr^2}{(r+h)^3} = 0. \quad (30)$$

Eq. (30) is a cubic equation.

$$r^3 + (3h - 2M)r^2 + 3h^2r + h^3 = 0. \quad (31)$$

The coefficient of the term  $r^3$  is greater than zero; the cubic equation has three roots. This article only discusses the solutions when  $r > 0$ .

According to its discriminant,  $-24M^3h^3 + 81M^2h^4$ , we derived the following:  $M > M_* \equiv \frac{27h}{8}$ .  $f_{\text{CH}}(r) = 0$  allows two real roots; however,  $M = M_*$

only contains one real root. These are future external and internal trap horizons surrounding the gravitational trapping region, as illustrated in Fig. 1.

To derive exact solutions for the two horizons ( $r_+$  and  $r_-$ ), we defined the following:

$$\cos \theta = \frac{\left(1 - \frac{9h}{2M} + \frac{27h^2}{8M^2}\right)}{\left(1 - \frac{9h}{M} + \frac{27h^2}{M^2} - \frac{27h^3}{M^3}\right)^{\frac{1}{2}}}. \quad (32)$$

The analytical solution of  $r > 0$  in Eq. (31) can be expressed as follows:

$$\begin{aligned} r_+ &= -h + \frac{2}{3}M \left(1 + 2\sqrt{1 - \frac{3h}{M} \cos\left(\frac{\theta}{3}\right)}\right), \quad (33) \\ r_- &= -h + \frac{2}{3}M \left(1 - 2\sqrt{1 - \frac{3h}{M} \cos\left(\frac{\theta + \pi}{3}\right)}\right). \end{aligned} \quad (34)$$

At one limit,  $M = M_*$ ,  $\theta = \pi$ , and  $\cos \theta = -1$ . Under this condition, the two horizons merged into one at the following:

$$r_+ = r_- = r_* = \frac{16}{27}M_* = 2h. \quad (35)$$

Another interesting limit was found at  $M \gg h$ . Under this condition,  $\theta = 0$  and  $\cos \theta = 1$ .

$$\begin{aligned} r_+ &= -h + \frac{2}{3}M \left(1 + 2\sqrt{1 - \frac{3h}{M}}\right) \\ &= 2M - 3h - \frac{3h^2}{2M} - \frac{9h^3}{4M^2} + O(h^4) \cong 2M, \end{aligned} \quad (36)$$

$$\begin{aligned} r_- &= -h + \frac{2}{3}M \left(1 - 2\sqrt{1 - \frac{3h}{M}}\left(\frac{1}{2}\right)\right) \\ &= \frac{3h^2}{4M} + \frac{9h^3}{8M^2} + \frac{135h^4}{64M^3} + O(h^5) \cong \frac{3h^2}{4M}, \end{aligned} \quad (37)$$

where the  $r_+$  horizon is approximated to  $2M$ , the horizon of Schwarzschild's metric with a mass of  $M$ . The  $r_-$  horizon is approximated to  $\frac{3h^2}{4M}$ , which has a positive value close to zero.

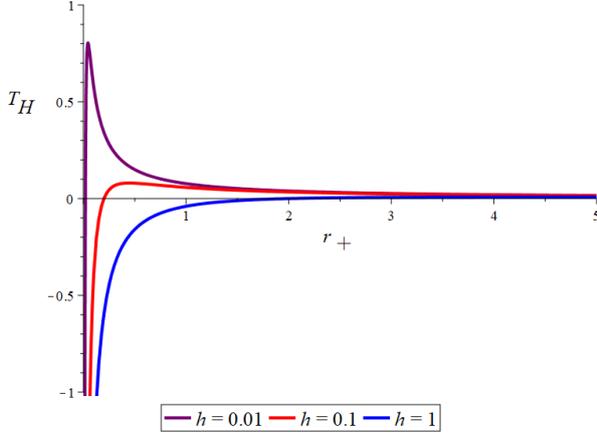


Figure 2: Plot of the Hawking radiation temperature as a function of  $r_+$ . We let  $h = 1$  (blue),  $0.1$  (red), and  $0.01$  (purple).  $T_H$  has a maximum value of  $\frac{5-2\sqrt{6}}{4\pi h}$  when  $r_+ = (2 + \sqrt{6})h$ . It then quickly becomes 0 when  $r_+ = 2h$ , turning negative when  $r_+ < 2h$ .

## VI Hawking radiation

Hawking radiation is a quantum effect of black holes, in which the quantum tunneling effect causes particles in black holes to pass through the event horizon. The tunneling probability of this process can be calculated. We do not discuss the derivation process in detail here in this paper. The results indicate that the Hawking radiation is proportional to the gravity  $\kappa$  on the horizon surface. The Hawking radiation temperature ( $T_H$ ) for metric (22) can be expressed as follows:

$$T_H \equiv \frac{\kappa}{2\pi} = \frac{f'_{\text{CH}}(r_+)}{2}. \quad (38)$$

The results of Eq. (21) were inserted into Eq. (38), and the following equation for  $T_H$  was derived:

$$T_H = \frac{2M(r_+^2 - 2hr_+)}{4\pi(r_+ + h)^4} = \frac{(r_+ - 2h)}{4\pi r_+(r_+ + h)}, \quad (39)$$

where  $T_H$  is a function of  $r_+$ . We let  $h = 1, 0.1$ , and  $0.01$  to plot a function graph of  $T_H$  versus  $r_+$ , as shown in Fig. 2. It shows that when  $r_+$  is close to 0, unlike the  $T_H$  of the Schwarzschild metric, the  $T_H$  has a maximum value of  $\frac{5-2\sqrt{6}}{4\pi h}$  when  $r_+ = (2 + \sqrt{6})h$ , and then quickly becomes 0 when

$r_+ = 2h$ , and turns negative when  $r_+ < 2h$ . In addition, we can elucidate Hawking radiation temperature by observing two limits. At one of the limits,  $M = M_*$  and  $r_+ = 2h$ , where the  $T_H$  approximates zero. Therefore, the proposed model predicts that radiation ceases but does not completely evaporate when the mass of the black hole reaches the critical value  $M_*$ . Naturally, the other limit was at  $M \gg h$ . At this instance,  $r_+ \cong 2M$ , whereby the  $T_H$  approximated to Schwarzschild's metric,  $T_H \cong \frac{1}{8\pi M}$ .

## VII Asymptotic behavior and quantum correction

We find from the asymptotic behavior of this singularity-free metric that there are several noteworthy characteristics. It approaches a static, spherically symmetric charged black hole satisfying Einstein–Maxwell equations and meets the quantum correction under the effective field theory. First, the Taylor expansion of the Chou-Huang function approximating the center can be expressed as follows:

$$\begin{aligned} f_{\text{CH}}(r) &= 1 - \frac{2Mr^2}{h^3} + \frac{6Mr^3}{h^4} + O(r^4) \\ &\cong 1 - \frac{2GMr^2}{c^2h^3}, \end{aligned} \quad (40)$$

where all the physical constants were regressed. Subsequently, de Sitter's spacetime can be expressed as follows:

$$f_{\text{ds}}(r) = 1 - \frac{\Lambda}{3}r^2. \quad (41)$$

This equation is like that of Hayward's spacetime. The de Sitter's core protected spacetime from the presence of singularity. We compared Eqs. (40) and (41) and found several interesting interactions between the physical constants.

$$\Lambda \cong \frac{6GM}{c^2h^3}. \quad (42)$$

Therefore, the singularity-free physical characteristics of  $h$  are associated with the cosmological constant  $\Lambda$ .

Moreover, when  $r \rightarrow \infty$ , this metric asymptotically approximates to the following Taylor expansion:

$$-g_{tt} = 1 - \frac{2M}{r} + \frac{6Mh}{r^2} - \frac{12Mh^2}{r^3} + O\left(\frac{1}{r^4}\right), \quad (43)$$

where the  $r^{-1}$  term can be used to determine the association between  $M$  and the configured mass, the  $r^{-2}$  term can be used to determine the association between  $h$  and certain ‘‘Coulomb’’ charges, such as those in the Reissner–Nordström solution. We insert  $\alpha = \frac{h^4}{2Q_m^2}$ ,  $M = \beta \frac{h^3}{\alpha}$ , and  $\beta = 1/12$  into Eq. (43) and obtain the following:

$$-g_{tt} = 1 - \frac{2M}{r} + \frac{Q_m^2}{r^2} - \frac{12Mh^2}{r^3} + O\left(\frac{1}{r^4}\right), \quad (44)$$

where  $Q_m$  is the total magnetic charge. Metric (22) was asymptotically approximated to the Reissner–Nordström solution, a static spherically symmetrical charged black hole.

Furthermore, we found that the  $r^{-3}$  term can serve as a ‘‘quantum correction’’ factor in metric (22). Literature suggests that metrics must meet the ‘‘one-loop quantum correction’’ of Newtonian potential, derived from effective field theory, to effectively simulate quantum effect [25–28]. Specifically,

$$\Phi(r) = -\frac{GM}{r} \left(1 + \gamma \frac{l^2}{r^2}\right) + \frac{GQ_m^2}{2r^2} + O\left(\frac{1}{r^4}\right), \quad (45)$$

where  $G$  is the Newtonian constant of gravity,  $\gamma = \frac{41}{10\pi}$  [25],  $\gamma = \frac{121}{10\pi}$  [27], and  $l$  is the Planck length. The Newtonian limit for the standard Schwarzschild’s metric can be expressed as follows:

$$\Phi(r) = -\frac{1}{2}(1 + g_{tt}). \quad (46)$$

Equation (44) can be rewritten to restore the Newtonian constant of gravity. Thereafter, the following was obtained:

$$\Phi(r) = -\frac{GM}{r} \left(1 + \frac{6h^2}{r^2}\right) + \frac{GQ_m^2}{2r^2} + O\left(\frac{1}{r^4}\right). \quad (47)$$

A comparison of the coefficients revealed the relationship between  $h$  and  $l$ :

$$h = \sqrt{\frac{\gamma}{6}}l, \quad (48)$$

where  $h \sim 10^{-35}m$  is in the same order of magnitude as the Planck length.

## VIII Conclusion

This study proposes a novel spherically symmetric regular black hole solution. It was extended from our singularity-free Newtonian gravity, in which Ricci scalar and Ricci curvature invariant does not diverge as  $r \rightarrow 0$ . We prove that the physical meaning of  $h$  can be interpreted as magnetic monopole charges described in NED. The energy–momentum tensors of this source satisfy weak energy conditions. Under weak field limits, the Lagrangian density regresses to normal Maxwell’s equations. The asymptotic behavior of the metric shows that it has the de Sitter’s core in the center. Moreover, when  $r$  tends to infinity, it regresses to the Reissner–Nordström solution in the  $r^{-2}$  term and meets the quantum correction in the  $r^{-3}$  term. The above-mentioned results can be derived directly from our model without additional corrections, outperforming those produced by the Bardeen and Hayward models. It requires further investigations.

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