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Electric reduced transition probabilities of ^{186}W and ^{186}Os isobars through the interacting boson model-I

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In this study, we applied the Interacting Boson Model (IBM-I) to compute the electric reduced transition probabilities B(E2)↓ of even-even neutron rich ^{186}W and ^{186}Os isobars. The ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ has also been calculated for those isobars and the SU(3) symmetry for those isobars has been reported. $E(4_1^+)$ and $E(2_1^+)$ indicate the energy level of 4_1^+ and 2_1^+, respectively. We have described the strength of B(E2) in W.u. for ^{186}W and ^{186}Os isobars of some of the low-lying quadrupole collective states in contrast to obtainable measured data. The electric reduced transition probabilities B(E2)↓ from yrast state gamma transition from $12_1^+ \to 10_1^+$, $10_1^+ \to 8_1^+$, $8_1^+ \to 6_1^+$, $6_1^+ \to 4_1^+$, $4_1^+ \to 2_1^+$ and $2_1^+ \to 0_1^+$ and other bands states and B(E2) ratio of ^{186}W and ^{186}Os isobar have been compared with obtainable measured data and other previous studies. Also calculated were the systematic strength of B(E2), intrinsic quadrupole moments, and deformation parameters of even-even ^{186}W and ^{186}Os isobars. The data from these calculations are in good matching with the obtainable measured data. The IBM-I model for the strength of B(E2) has been systematically deduced in SU(3) limit for a few yrasts states transitions in ^{186}W and ^{186}Os isobars.

I Introduction

The interacting boson model (IBM-I) is a hypothetical framework used to describe the collective behaviour of atomic nuclei and was initially developed by Iachello and Arima [1–3]. In IBM-1, no distinction is made between proton pairs and neutron pairs, and s stands for bosons with angular momentum zero, while d stands for bosons with angular momentum two. The simple form of IBM-1 describes the low-lying collective excitations of an even-even nucleus in terms of the s($L = 0$) and d($L = 2$) bosons [3]. This model delivers a modest and intuitive manner of describing the properties of even-even nuclei, such as the W and Os isobars with even neutrons $N = 112$ and 110, respectively.

In the IBM-I, the B(E2) value represents the probability of an electromagnetic transition be-

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tween two states in a nucleus, specifically the quadrupole transition strength. The B(E2) value is related to the intrinsic quadrupole moment of a state and provides information about the shape and collectivity of the nucleus and gives explanation of electromagnetic transition rates in medium mass nuclei. In the first estimation, only pairs with s-bosons and d-bosons, where angular momentum $L = 0$ and $L = 2$, respectively, are considered. The IBM-1 model is related through a characteristic group structure, which indicate three types symmetries $U(5)$, $SU(3)$ and $O(6)$ [4,5].

The ^{186}W nucleus consists of protons $Z = 74$, and neutrons $N = 112$. The ^{186}Os nucleus consists of $Z = 76$ and $N = 110$. In recent years, even-even isotopes of Tungsten W ($Z = 74$) and Osmium Os ($Z = 76$) had been investigated extensively, both theoretically and experimentally [6–8]. Al-Jubbori et al. [9] studied nuclear structure of rare-earth ^{172}Er, ^{174}Yb, ^{176}Hf, ^{178}W, ^{180}Os nuclei. Kassim et al. [10] studied negative parity low spin states of even-odd $^{187\text{-}197}$Pt isotopes. Yrast states of up to $\mathrm{I}^{\pi} = 12^+$ in ^{186}W ($Z = 74, N = 112$) and ^{186}Os ($Z = 76, N = 110$) isobars have been found by π $\mathrm{h}^{-8}{}_{11/2}$ $\mathrm{i}^{-14}{}_{13/2}$ configurations for double magic $Z = 82, N = 126$ in ^{208}Pb nucleus. The quadrupole E2 transitions incur collective properties in ^{184}W and ^{184}Os isobars, which had been studied both theoretically and experimentally [6–8]. It is interesting to observe the collective properties of nuclei far away from the closed shell of the ^{208}Pb nucleus, where the double magic numbers $Z = 82$ and $N = 126$ found a closed shell in ^{208}Pb nucleus. Studies of the systematic collective properties of ^{184}W and ^{184}Os isobars by the IBM-1 model suggest investigation into the collective properties in higher mass isobar $A = 186$ in ^{186}W and ^{186}Os nuclei. The present work is a gorgeous challenge to study the systematic collective nature of B(E2) strength in ^{186}W and ^{186}Os isobars by IBM-1. Recently, Radhi et al. [11] described energy levels and decay properties in ^{158}Gd nucleus via IBM model. We reported the O(6) symmetry in ^{108,110,112}Ru isotopes by IBM-1 calculation [12]. The theoretical works discussing intruder configuration and configuration mixing by means of IBM-I around shell closure $Z = 82$ had been discussed in the literature on multiple occasions [13–20]. Using IBM-1, the B(E2) values of even-even $^{104\text{-}112}$Cd isotopes were carried out by Abdullah et al. [21]. The B(E2) values of even mass $^{102\text{-}106}$Pd isotopes[22], and $^{108\text{-}112}$Pd[23] and $^{104\text{-}112}$Cd[24] isotopes were carried out using the IBM-1 model.

In the present work we focus on evaluating the deformation parameters, intrinsic quadrupole moments, and strength of B(E2) values and ratios of B(E2) of yrast state bands, γ-band and β-bands of ^{186}W and ^{186}Os nuclei up to 12^+ to 10^+ levels through E2 transition. Given the large amount of data that has collected over the past twelve years, it is of interest to further explore the above mentioned properties of ^{186}W and ^{186}Os nuclei. For this reason, we would like a widespread analysis of the low-lying structure of even-even ^{186}W and ^{186}Os isobars by IBM-I and to focus particularly on B(E2) systematics, ratio of B(E2), intrinsic quadrupole moments, deformation parameters, and the role of SU(3) symmetry for yrast states of E2 transition in ^{186}W and ^{186}Os isobars.

II Theoretical calculation

i Calculation of B(E2) in IBM-1 model

The excitation level of even-even nuclei for the yrast state bands are $\mathrm{L_i} = 2^+, 4^+, 6^+, 8^+, 10^+ \ldots\ldots$ and they become lower states ($\mathrm{L_f} = \mathrm{L_i}$ -2) in the gamma transition E(2). The E2 transition with the corresponding strength of B(E2) is referred to as having reduced transition probabilities. The B(E2) value in SU(3) symmetry is usually calculated by IBM-1 according to following expression [25]:

$$B(E2; L+2 \to L)_{\downarrow} = \left(\frac{3}{4}\right) \frac{(L+3+2N)}{(2L+5)(L+3)} \alpha_2^2 \\ (L+1)(L+2)(2N-L) \quad (1)$$

where symbol N is boson number, which is equal to half the number of valence nucleons (proton and neutrons) in double magic nuclei ^{208}Pb, L is the state that nucleus translates to $L-2$ and α_2^2 specifies the square of effective charge. The B(E2) ratio is calculated using the following formulas [3]:

$$\frac{B(E2; g \to g)}{B(E2; 2_g^+ \to 0_g^+)} = \frac{10}{7} \approx 1.4 \quad (2)$$

After first calculating the effective charge from the given experimental value B(E2) of transition

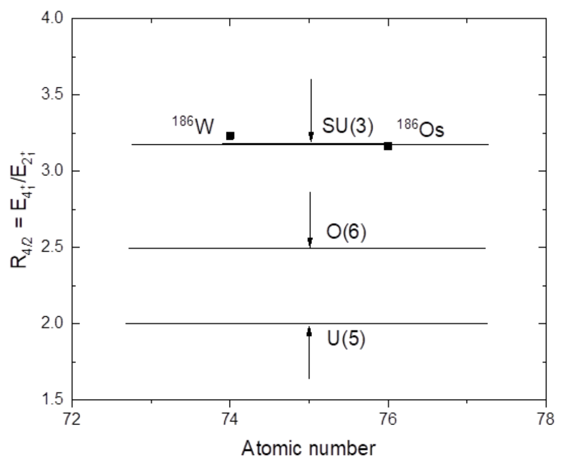

Figure 1: $E(4_1^+)/E(2_1^+)$ Vs number of protons of even-even ^{186}W, ^{186}Os isobars. $E(2_1^+)$ indicate $1^{st}2^+$ level and $E(4_1^+)$ indicate $1^{st}4^+$ level. Solid line represented by error indicate U(5), O(6) and SU(3) limit.

($2^+ \to 0^+$), one can calculate the value of parameter α_2^2 for each isotope. The effective charge is useful to calculate the B(E2) transition from $L+2 \to L$, i.e in the present case from 12^+ to 10^+, 10^+ to 8^+, 8^+ to 6^+, 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+, and other transitions from γ and β bands.

ii Quadrupole moments

The quadrupole moments give information about deformation regarding prolate or oblate shape. The quadrupole moments (Q_0 and ($Q_{2_1^+}$) can be derived [3, 25] as

$$Q_0 = \alpha_2 \left(\frac{16\pi}{40}\right)^{1/2} (4N+3) \tag{3}$$

$$Q_{2_1^+} = \alpha_2 \left(\frac{16\pi}{40}\right)^{1/2} \frac{2}{7}(4N+3) \tag{4}$$

iii Deformation parameters

The deformation parameters are calculated from upword B(E2)↑ transition 0_1^+ to 2_1^+ which is related to β^2 [26] as

$$B[E2; 0^+ \to 2^+] \uparrow = [(3/4\pi)eZR_0^2]^2\beta^2 \tag{5}$$

Quadrupole deformation parameters β are calculated [26] as

$$\beta = [3ZeR_0^2/4\pi]^{-1}[B(E2)\uparrow]^{1/2} \tag{6}$$

R_0 is the average radius of nucleus, Z is the atomic number, e is charge (1.6x10^{-19}coul) of electron and the radius is related to mass number A in the following equation (7):

$$R_0^2 = 0.0144A^{2/3}b \tag{7}$$

Here, $b(1barn = 10^{-28}\ \text{m}^2)$ is a unit of area.

III Results and discussion

i $R_{4/2}$ classifications for symmetry

The collective energies of even-even nuclei consist of three groups based on the ratio $R_{4/2}$ of excitation energies of the first 4_1^+ and first 2_1^+ excited states:

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)} \tag{8}$$

where $E(4_1^+)$ is the energy level at 4_1^+ and $E(2_1^+)$ is the energy level at 2_1^+. Even-even nuclei can be classified according to ratios $R_{4/2}$ [27]. That $R_{4/2} = 2.00$ indicates a harmonic vibrator U(5), $R_{4/2} = 2.50$ indicates γ-unstable O(6) symmetry and $R_{4/2} = 3.33$ indicates an axially symmetric rotor SU(3) [27]. We identified SU(3) symmetry for even-even nuclei with $A = 186$ isobars in ^{186}W and ^{186}Os nuclei since $R_{4/2}$ values of both nuclei are 3.234 [27, 28], and 3.165 [27, 28], respectively presented in Table 1. The ratio of E(4_1^+)/E(2_1^+) values as a function of even atomic numbers of ^{186}W and ^{186}Os isobars is shown in Fig. 1. The solid line shows U(5), O(6) and SU(3) symmetry [27,28]. The solid line SU(3), near the ratios of $R_{4/2}$ values of ^{186}W and ^{186}Os isobars, [27, 28] is presented in Fig. 1.

Table 1: The ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$ for even-even ^{186}W and ^{186}Os nuclei.

Nucleus	^{186}W	^{186}Os
$R_{4/2}$	3.234	3.165

Table 2: Comparison of the calculated B(E2) ratios with previous studies(Th.) for ^{186}W and ^{186}Os nuclei.

Nucl.	N	B(E2) ratios	Exp.[27]	IBM-1	Th. [17, 18]
^{186}W	$4+7=11$	$4_1^+ \to 2_1^+/2_1^+ \to 0_1^+$	1.39(12)	1.402	1.49
		$6_1^+ \to 4_1^+/2_1^+ \to 0_1^+$	1.69(12)	1.493	1.76
		$8_1^+ \to 6_1^+/2_1^+ \to 0_1^+$	1.60(12)	1.462	1.99
		$10_1^+ \to 8_1^+/2_1^+ \to 0_1^+$	1.36(36)	1.421	2.20
^{186}Os	$3+8=11$	$4_1^+ \to 2_1^+/2_1^+ \to 0_1^+$	1.45(7)	1.402	1.50
		$6_1^+ \to 4_1^+/2_1^+ \to 0_1^+$	1.99(7)	1.493	1.80
		$8_1^+ \to 6_1^+/2_1^+ \to 0_1^+$	1.89(11)	1.485	2.11
		$10_1^+ \to 8_1^+/2_1^+ \to 0_1^+$	2.06(44)	1.421	2.45

ii Boson number

A boson such as s($l=0$) or p($l=1$) is a pair of valence nucleons (protons and neutrons), while a boson number is calculated as the quantity of combined pairs of valence nucleons. The valance nucleons are calculated from double mirror nuclei. In the present study the valance nucleons are counted from double magic nucleus ^{208}Pb ($Z=82, N=126$). The number of valance protons $\mathrm{N_p}$ and neutron $\mathrm{N_n}$ has a total $N=(N_p+N_n)/2=(n_\pi+n_\nu)$ bosons. The ^{208}Pb doubly magic nucleus is taken as an inert core to find the boson number, and the total number of bosons of both ^{186}W and ^{186}Os nuclei are $(8/2+14/2)=11$ and $(6/2+16/2)=11$, respectively, which is presented in Table 2.

iii Systematic reduced transition probabilities: B(E2) and B(E2) ratios

In order to obtain reduced transition probability values, first we have calculated value of effective charge (α_2) with IBM-I. The effective charges have been estimated by normalizing the measured data $B(E2; 2_1^+ \to 0_1^+)$ of each isobar using Eq. (1). Using the value of effective charge, we have calculated B(E2) values from transitions $4_1^+ \to 2_1^+$,$6_1^+ \to 4_1^+$,$8_1^+ \to 6_1^+$,$10_1^+ \to 8_1^+$,$12_1^+ \to 10_1^+$ and other transitions from γ and β bands. Values of fitted parameter (α_2^2) in W.u. with error estimation and boson numbers and B(E2) values for ground states, γ-band and β-band transitions are presented in Table 3. Fig. 2 shows the theoretical and previous measured data of B(E2) in W.u. plotted as a function of spin for both isobars. The square of effective charge (α_2) of ^{186}W and ^{186}Os isobars are 2.018(035) W.u. and 1.678(042) W.u, respectively. In Table 3 and Fig. 2 is shown the ground state transition of B(E2) strength of both isobars as compared with previous available measured data of B(E2), the present work by IBM-1 and previous available theoretical (Th) data in W.u. As is clear from Table 3 and Fig. 2, the reduced transition probabilities, calculated using IBM-I and measured data are consistent with each other. Also, B(E2) values increase almost linearly with the increase up to spin 6^+ for both isobars.

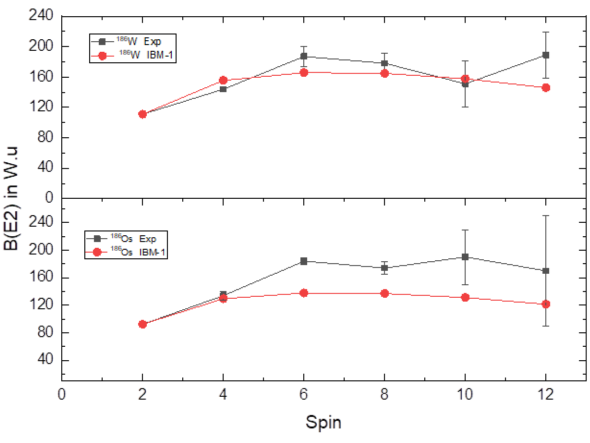

Figure 2: $B(E2; I_i \to I_f)$ values versus yrast transition of even ^{186}W, ^{186}Os isobars.

Table 3: Reduced transition probability B(E2)↓ in even even ^{186}W and ^{186}Os nuclei.

Nucl.	α_2^2 W.u	N	Transition level	B(E2) exp. W.u	B(E2) IBM-1 W.u	Th[29]
^{186}W	2.018 ± 0.035	$4+7=11$	$2_1^+ \to 0_1^+$	112.4 ± 15	112.03	82.063
			$4_1^+ \to 2_1^+$	$144(\pm 11 - 10)$	157.14	117.77
			$6_1^+ \to 4_1^+$	$181(+15-13)$	167.30	128.57
			$8_1^+ \to 6_1^+$	$178(+13-12)$	163.81	128.57
			$10_1^+ \to 8_1^+$	$152(+18-34)$	159.20	126.98
			$12_1^+ \to 10_1^+$	$191(+22-45)$	155.55	—
			$2_2^+ \to 2_1^+$	$10.1(\pm 7)$	Forbidden	15.55
			$2_2^+ \to 4_1^+$	$1.9(+12-10)$	Forbidden	—
			$2_2^+ \to 0_1^+$	$4.35(+28-26)$	Forbidden	1.65

Nucl.	α_2^2 W.u	N	Transition level	B(E2) exp. W.u	B(E2) IBM-1 W.u	Th[30, 31]
^{186}Os	1.678 ± 0.042	$3+8=11$	$2_1^+ \to 0_1^+$	93.6 ± 21	93.52	100
			$4_1^+ \to 2_1^+$	135 ± 7	131.17	141.26
			$6_1^+ \to 4_1^+$	$185(+8-5)$	139.666	—
			$8_1^+ \to 6_1^+$	132 ± 7	138.92	—
			$10_1^+ \to 8_1^+$	190 ± 60	132.90	—
			$12_2^+ \to 10_1^+$	$166(+23-132)$	122.96	—
			$2_2^+ \to 2_1^+$	22.1 ± 17	Forbidden	0.0317
			$2_2^+ \to 4_1^+$	1.2 ± 4	Forbidden	—
			$2_2^+ \to 0_1^+$	9.4 ± 8	Forbidden	0.1111
			$0_2^+ \to 2_1^+$	0.066	Forbidden	—
			$4_2^+ \to 2_2^+$	$69(+11-13)$	75.23	—
			$4_2^+ \to 4_1^+$	$24(+4-5)$	Forbidden	—
			$4_2^+ \to 2_1^+$	$3.0(+5-6)$	Forbidden	—
			$4_2^+ \to 4_2^+$	$17(+8-9)$	0.476	—
			$4_3^+ \to 3_1^+$	$45(+10-15)$	93.65	—
			$4_3^+ \to 2_2^+$	27 ± 9	15.87	10.9
			$6_2^+ \to 4_2^+$	$120(+30-50)$	95.23	—
			$6_2^+ \to 6_1^+$	7.2	Forbidden	—
			$6_2^+ \to 4_1^+$	$1.2(+4-5)$	Forbidden	—
			$8_2^+ \to 6_2^+$	99 ± 18	77.46	—

Further results obtained in this present work include the calculations of B(E2) values as compared with previous studies (Th.), and they are presented in Table 3. This comparison shows that the calculated B(E2) values are better than those in [29–31]. Therefore, the even-even ^{186}W, ^{186}Os isobars were reproduced nicely by previous experimental data and fit satisfactorily with the theoretical data, except the transitions $6_1^+ \to 4_1^+$ and $4_3^+ \to 4_2^+$ in the ^{186}Os nucleus. The calculated results are in good agreement with previously reported measured data [27, 28]. Furthermore, the transitions from γ and β bands to the ground band is forbidden in the O(6) dynamical symmetry according to the selection rules $\Delta\lambda = 0$, $\Delta\mu = 0$ [3, 4].

In fact, in IBM-I, both proton and neutron bosons are indistinguishable as long as the valence protons and neutrons are existing as either both hole-like or both particle-like in nature. For each of the studied nuclei, the B(E2) ratio is determined using Eq.(2) and presented in Table 2. The IBM-1 calculations of B(E2) ratios are compared with experimental data and previous studies (Th.) [17, 18]. From Table 2, the results of the calculations align nicely with experimental data, better than those in the previous theoretical studies.

Table 4: Quadrupole moment (barn) and deformation parameter of even ^{186}W, ^{186}Os nuclei.

Nucl.	β_{exp}	β_{cal}	Q_{0exp}	Q_{0cal}	Q_{2exp}	Q_{2cal}
^{186}W	0.226 ± 0.039	0.225 ± 0.029	5.39 ± 0.10	5.94 ± 0.78	—	—
^{186}Os	0.200 ± 0.003	0.235 ± 0.027	5.40 ± 0.09	5.414	-1.60	-1.55

iv Quadrupole moments and the deformation parameter (β)

The quadrupole moment (Q) is a property used to describe the distribution of electric charge within a nucleus. It provides information about the shape of an object and is related to the spatial asymmetry of its charge distribution. In the context of electric charge, the quadrupole moment describes the deviation from a spherically symmetric charge distribution. It characterizes how the charges are distributed in the nucleus, giving information about its shape, as spherical, deformed oblate or prolate in shape. Intrinsic quadrupole moments of Q_0 and Q_2 are calculated using Eq. (3) and Eq. (4) for even-even ^{186}W and ^{186}Os isobars and are presented in Table 4. Intrinsic quadrupole moments are simply calculated from B(E2) values using restrictive assumption of a rigid shape of a nucleus and compared with the values given in the previous literature [27, 28], which confirms that the results obtained in the current study is in good agreement with the available previously measured data. Deformation parameters (β) of nuclei with even mass $A = 186$ isobars in ^{186}W and ^{186}Os are computed using Eq. (5) and Eq. (6) and displayed in Table 4. Calculated deformation parameters using preventive hypothesis of a rigid shape of a nucleus have been matched by means of those given in [27, 28]. In Table 4, it is clear that calculated deformation parameters, and intrinsic quadruple moments are consistent with the available measured data. Figure 3 shows the quadrupole moments and deformation parameter as a function of atomic number. It is shown that the quadrupole and deformation parameter for both isobars decreases as the atomic numbers are increased from 74 to 76. Calculations obtained in the present study are consistent with those given in previous literature [27, 28].

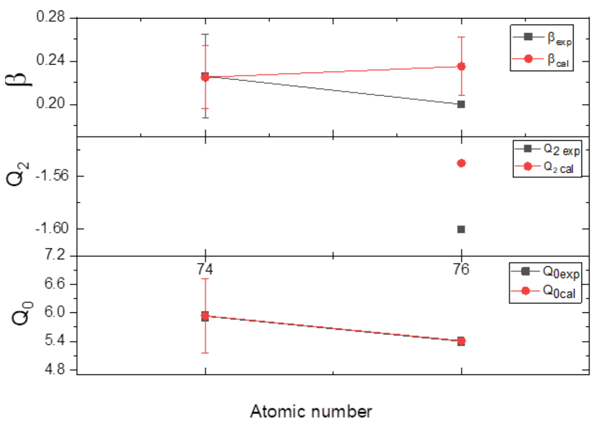

Figure 3: Quadrupole moments and deformation parameter versus number of protons of even-even ^{186}W, ^{186}Os isobars.

IV Conclusions

The values of electric quadrupole strength B(E2)↓ and B(E2) ratios are reported for the yrast band from $2_1^+ \rightarrow 0_1^+$, $4_1^+ \rightarrow 2_1^+$, $6_1^+ \rightarrow 4_1^+$, $8_1^+ \rightarrow 6_1^+$, $10_1^+ \rightarrow 8_1^+$, $12_1^+ \rightarrow 10_1^+$ and other transitions from γ and β bands of even-even ^{186}W, and ^{186}Os isobars by IBM-1. The present calculation B(E2)↓ and B(E2) ratio are compared with previous measurements and previous theoretical data. It is found that present results coincide nicely with previous experimental data and better than those in the previous theoretical data. The computed deformation parameters and quadrupole moments were consistent with the available previous measured data. The analytic $R_{4/2}$ values for even-even nuclei with $A = 186$ isobars of ^{186}W and ^{186}Os nuclei are 3.234 and 3.165, respectively, which indicates SU(3) symmetry. The IBM-I calculation of B(E2) values of ^{186}W and ^{186}Os isobars has been performed in SU(3) character.

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[1] F Iachello, A Arima, *Boson symmetries in vibrational nuclei*, Phys. Lett. B **53**, 309, (1974).

[2] A Arima, F Iachello, *Collective nuclear states as representations of a SU(6) group*, Phys. Rev. Lett. **35**, 1069, (1975).

[3] F Iachello, A Arima, *The interacting Boson Model*, Cambridge Univ Press, Cambridge (1987).

[4] RF Casten, DD Warner, *The interacting Boson approximation*, Rev Mod Phys **60**, 389-469(1988).

[5] F Iachello, P Van Isacker, *The interaction Boson-Fermion Model*, Cambridge Univ Press, Cambridge, (1991).

[6] M K Muttaleb, *Nuclear deformation shape in rich neutrons $^{172\text{-}194}$Os isotopes*, J Kerbala Univ **10**, 33-40 (2012).

[7] F I Sharrad, H Y Abdullah, N Al-Dahan, et al., *Low lying states of ^{184}W and ^{184}Os nuclei*, Chin Phys C **37**, 034101(2013).

[8] I Hossain, H A Kassim , F I Sharrad, A S Ahmed, *Nuclear structure of yrast bands of ^{180}Hf, ^{182}W and ^{184}Os nuclei by means of interacting boson model-1*, ScienceAsia **42**, 22-27 (2016).

[9] M A Al-Jubbori, H H Kassim, E M Raheem , I M Ahmed, et al., *Nuclear structure of rare-earth ^{172}Er, ^{174}Yb, ^{176}Hf, ^{178}W, ^{180}Os nuclei*, UKR, J. Phys. Vol. **67**, 2 127-135(2022).

[10] H H Kassim, F I Sharrad, *Negative parity low spin states of even-odd $^{187\text{-}197}$Pt isotopes*, Nucl Phys A **380**, 365 (2015).

[11] F S Radhi, H H Kassim, M A Al-Jubbori, et al., *Description of energy levels and decay properties in ^{158}Gd nucleus*, Nucl. Phys. At. Energy **24**,3 (2023).

[12] I Hossain, H H Kassim, M A Al-Jubbori, et al., *Study of O(6) symmetry in ^{108,110,112}Ru isotopes by IBM-1 calculation*, Prob. of Atom Sci. Tech **3**(145)79 (2023)..

[13] A Kucukbursa, A Yoruk, *IBM-2 calculations on the some even-even tellurium isotopes*, Bulletin of Pure and Applied Sciences, **18D**, 177,1999.

[14] M Deleze et al., *Systematic study of the mixed ground-state and 'intruder' bands in ^{110,112,114}Cd*, Nucl. Phys. A **551**(2) 269 (1993).

[15] P E Garrett, K L Green, J L Wood, *Breakdown of vibrational motion in the Isotopes $^{110\text{-}116}$Cd*, Phys. Rev. C **78**, 044307 (2008).

[16] P Van Isacker, S Pittel, A Frank, P D Duval, *Microscopic study of configuration mixing in the interacting boson model*, Nucl. Phys. A **451**,2 202 (1986).

[17] I Inci, D Bonatsos, I Boztosunl, *Electric quadrupole transitions of the Bohr Hamiltonian with the Morse potential*, Phys. Rev. C **84** 024309 (2011).

[18] D Bonatsos, P E Georgoudis, D Lenis, et al., *Bohr Hamiltonian with a deformation-dependent mass term for the Davidson potential*, Phys. Rev. C **83** 044321 (2011).

[19] L Prochniak, P Quentin, M Imadalou, *Collective properties of stable even-even Cd isotopes*, Int. J. Mod. Phys. E **21**(5) 1250036 (2012).

[20] I M Ahmed et al., *The evolution properties of even-even $^{100\text{-}110}$Pd nuclei*, Int. J. Mod. Phys. E **21**(12) 1250101 (2012).

[21] H Y Abdullah, et al., *Electromagnetic reduced transition properties of even-even $^{104\text{-}112}$Cd isotopes*, Indian J. Phys. **87**, 571 (2013).

[22] I Hossain et al., *Electromagnetic reduced transition properties of the ground state band of even-even $^{102\text{-}106}$Pd isotopes by means of interacting boson model-1*, Ind. J. Phys. **88**(1), 5-9 (2014).

[23] I Hossain, H Y Abdullah, M I Ahmed, M A Saeed, *B(E2) value of even-even $^{108\text{-}112}$Pd isotopes by interacting boson model-I*, Chin. Phys. C Vol. **38** (2) 024103 (2014).

[24] I Hossain, M I Ahmed, F I Sharrad, et al., *Yrast states and B(E2) values of even $^{100\text{-}102}$Ru isotopes using interacting boson model (IBM-1)*. Ching Mai J. Sci. **42**(4), 1-10 (2015).

[25] O Scholten, F Iachello, *Interacting boson model of collective nuclear states III. The transition from SU(5) to SU(3)*, Annals Phys. **115** 325 (1978).

[26] S. Raman, Jr Nestor, P Tikkanen, *Transition probability from ground to the first-excited 2^+ state of even-even nuclides*, Atom Data Nucl. Data **78**, 1-128(2001).

[27] National Nuclear Data Center. Brookhaven National Laboratory. NY, USA.

[28] M C Baglin, *Nucl. Data sheets for A = 186*, Nucl. data sheets **99** (1-96) 2003.

[29] S Abu Musleh, O Scholten, *A description of odd mass W-isotopes in the Interacting Boson-Fermion Model*. Nuclear Physics A **878** 37 (2012).

[30] A A Phillips, P E Garrett, N Lo Iudice, A V Sushkov, et al., *Structure of the $K\pi = 4^+$ bands in ^{186,188}Os*, Phys. Rev. C **82**, 034321 (2010).

[31] A Bouldjedri, M L Benabderrahmane, *Broken SU(3) symmetry at low spin in $^{178-186}$Os*, J. Phys. G: Nucl. Part. Phys. **29**, 1327 (2003).